

# On the quantum loop suppressed electroweak processes

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## Abstract

The quantum loop suppressed electroweak processes appear to be very sensitive probes for the symmetry-breaking mechanisms. Since the Standard Model does not involve massive neutrinos, baryon or lepton number violations and the cold dark matter particle, there are great expectations in search for physics beyond the Standard Model. The LHC experiments reported no new physics, except for the probable appearance of the 125 GeV heavy resonance. If this resonance turns out to be a Higgs boson, we shall be faced with the paradox that the Standard Model, as a theoretically and experimentally incomplete and defective theory, is confirmed. If, on the other hand, this resonance is a pseudoscalar meson - a mixture of a pseudoscalar glueball and toponium, as suggested by P. Cea, one has to study alternative symmetry-breaking mechanisms. This paper is devoted to the evaluations of the gauge invariant electroweak functions that are relevant for the suppressed electroweak processes within the SM or the UV finite theory in which the noncontractible space breaks the gauge symmetry. Deviations from the SM amplitudes range 10% to 30%, thus being measurable at the LHCb and future superB factories.

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## I. INTRODUCTION AND MOTIVATION

The physics of suppressed electroweak (EW) processes by quantum loops figure as the best window for search into the new physics territory. More than three decades of the theoretical work strengthens our ability to predict their amplitudes with high precision. While electroweak and hard QCD calculations rely on the perturbative framework and the renormalization theory, the hadron matrix elements are most accurately evaluated within the lattice QCD.

The extensive search for new physics at the LHC reveals a new boson particle [1] with a mass around 125 GeV. Since the Standard Model (SM) does not contain massive neutrinos, lepton or baryon number violations and a candidate particle for the cold dark matter, it is impossible that the new particle could be the SM Higgs boson. However, any extension of the SM that has to fulfil above improvements, such as SUSY or GUT extension, implicates the existence of more scalar particles. It is notorious that the Higgs mechanism does not solve the problem of particle masses but only breaks the gauge symmetry, i.e. the fermion masses are free parameters.

The alternative explanation of the 125 GeV resonance is its identification as the pseudoscalar meson, which is a mixture of the pseudoscalar glueball and toponium, suggested by P. Cea [2], thus leaving the problem of the symmetry-breaking mechanism unsolved.

Anyhow, the new LHC data with the total and partial decay widths of the new particle, as well as its mass, spin and parity, should resolve this puzzle. The first LHC results [1] point to a surplus of photons and a deficit of Z bosons in the decay of the 125 GeV resonance assumed to be a SM Higgs scalar, as well as to the absence of any decay signal into leptons.

In this paper, we present complete one loop results for the gauge invariant functions appearing in the suppressed EW processes within the SM and the BY theory [3], the theory without the Higgs scalar but with the Lorentz and gauge invariant UV cut-off. The next chapter is devoted to the explicit description of the calculations with details in the Appendix, while the concluding section provides numerical estimates and points out the difference between the SM and the BY theory.

## II. EVALUATION OF THE GAUGE INVARIANT FUNCTIONS

In this paper, we shall omit a detailed description of the BY theory with its mathematical [3] and phenomenological advantages in particle physics and cosmology [4–6]. The Lorentz and gauge invariant cut-off of the BY theory is fixed by the Wick's theorem and trace anomaly [3] through weak coupling and the weak gauge boson mass  $\Lambda = \frac{\hbar}{cd} = \frac{2}{g} \frac{\pi}{\sqrt{6}} M_W \simeq 326 GeV$  where  $g$ =weak coupling,  $M_W$ =weak gauge boson mass. The noncontractible space cannot essentially affect the strong coupling below the scale  $\mu \simeq 200 GeV$  [4], hence the impact of  $\Lambda$  on the hard QCD corrections to the electroweak processes [7] is negligible. Thus, we focus onto the dominant one loop electroweak contributions written in the form of the gauge invariant functions  $A, X, Y, Z, E$  [8]. These functions appear in the meson mixings  $M^0 - \bar{M}^0$  and meson decays  $M \rightarrow l^+ l^-$ ,  $M_1 \rightarrow M_2 \nu \bar{\nu}$ ,  $M_1 \rightarrow M_2 l^+ l^-$ ,  $M_1 \rightarrow M_2 \gamma$  [8–10].

The Inami-Lim functions  $S_0(x_t) = A(x_t)$  and  $Y(x_t)$  are treated and compared between the SM and BY theory in refs. [11] and [12]. We have to reevaluate the self-energy, vertex and box type diagrams for Z bosons, photons and gluons in figs. 1-4 of ref. [9] in both the SM and the BY theory to get the explicit forms for the gauge invariant  $X, Y, Z, E$  functions.

We list now our results obtained in the 't Hooft-Feynman gauge for the  $B, C, D, E$  functions [8] referring to figures 1-4 in ref. [9]:

$B_1$  box diagram as a sum of four graphs of Fig. 2 (a),(b),(c),(d) of ref. [9]:

$$B_1 = M_W^2 \left[ \frac{1}{4} f_1(m_l, m_\alpha, M_W) \left( 1 + \frac{1}{4} \frac{m_l^2}{M_W^2} \frac{m_\alpha^2}{M_W^2} \right) - \frac{1}{2} \frac{m_\alpha^2}{M_W^2} m_l^2 g_1(m_l, m_\alpha, M_W) \right], \quad (1)$$

$m_l$  is the lepton mass,  $m_\alpha$  is the quark mass and  $M_W$  the weak boson mass. For the definitions and analytic expressions of  $g_1$  and  $f_1$  functions, see the Appendix.

$B_2$  box diagram as a graph of Fig. 2 (a) of ref. [9]:

$$B_2 = -\frac{1}{4} M_W^2 L(M_W, m_\alpha), \quad (2)$$

for the definition and analytic expression of the L function, see the Appendix.

$C = \frac{1}{2} \Gamma_Z = \frac{1}{2} \sum_{i=(a)}^{(h)} \Gamma_Z^i$  vertex is given in Fig. 3 (a-h) of ref. [9] and is reevaluated in ref. [12]:

$$\begin{aligned}
\Gamma_Z^{(a+b)} &= \left(-\frac{1}{2} + \frac{1}{3}s_W^2\right)\left(1 + \frac{1}{2}\frac{m_j^2}{M_W^2}\right)\tilde{B}_1(0; m_j, M_W), \\
\Gamma_Z^{(c)} &= -\frac{1}{4}\left(-1 + \frac{4}{3}s_W^2\right)(\tilde{B}_0(0; m_j, M_W) + m_j^2 L(m_j, M_W)) + \frac{2}{3}s_W^2 m_j^2 L(m_j, M_W), \\
\Gamma_Z^{(d)} &= -\frac{1}{2}\frac{m_j^2}{M_W^2}\left(\frac{1}{3}s_W^2\tilde{B}_0(0; m_j, M_W) + m_j^2\left(\frac{1}{2} - \frac{1}{3}s_W^2\right)L(m_j, M_W)\right), \\
\Gamma_Z^{(e)} &= -\frac{3}{2}(1 - s_W^2)(\tilde{B}_0(0; m_j, M_W) + M_W^2 L(M_W, m_j)), \\
\Gamma_Z^{(f+g)} &= -s_W^2 m_j^2 L(M_W, m_j), \\
\Gamma_Z^{(h)} &= \frac{1}{8}(-1 + 2s_W^2)\frac{m_j^2}{M_W^2}(\tilde{B}_0(0; M_W, m_j) + M_W^2 L(M_W, m_j)),
\end{aligned}$$

where  $\tilde{B}_0$  and  $\tilde{B}_1$  Green functions are defined and evaluated in the Appendix ( $m_j$  denotes the quark mass and  $s_W^2 \equiv \sin^2(\Theta_W)$ ).

$D = F_1(q^2\gamma^\mu P_L) = F_1(-\not{q}q^\mu P_L)$  denotes the contribution to the induced quarks-photon vertex to the second order in the external momentum  $q$  from graphs of fig. 3 (a-h) of the ref. [9] (Eq. (B.1)). The form factors  $F_1$  extracted from the two different Lorentz structures,  $q^2\gamma^\mu P_L$  and  $-\not{q}q^\mu P_L$ , must be equal and this is a check on the validity of our calculation:

$$D = F_1(q^2\gamma^\mu P_L) = (a + b) + (c) + (d) + (e) + (f + g) + (h), \quad (3)$$

$$\begin{aligned}
(a + b) &= -\frac{1}{3}M_W^2\left(1 + \frac{1}{2}\frac{m_\alpha^2}{M_W^2}\right)\tilde{B}'_1(0; m_\alpha, M_W), \\
(c) &= \frac{4}{3}M_W^2[\tilde{C}_1(0; m_\alpha, M_W) + \tilde{C}_2(0; m_\alpha, M_W) + 2\tilde{C}'_3(0; m_\alpha, M_W) - m_\alpha^2\tilde{C}'_0(0; m_\alpha, M_W)], \\
(d) &= -\frac{2}{3}m_\alpha^2[m_\alpha^2\tilde{C}'_0(0; m_\alpha, M_W) - 2\tilde{C}'_3(0; m_\alpha, M_W) - \tilde{C}_1(0; m_\alpha, M_W) - \tilde{C}_2(0; m_\alpha, M_W)], \\
(e) &= M_W^2[-4\tilde{C}'_3(0; M_W, m_\alpha) - \frac{1}{2}\tilde{B}'_0(0; m_\alpha, M_W) - 2M_W^2\tilde{C}'_0(0; M_W, m_\alpha) \\
&\quad - \frac{1}{2}\tilde{C}_0(0; M_W, m_\alpha) - 2\tilde{C}_1(0; M_W, m_\alpha)], \\
(f + g) &= 2m_\alpha^2 M_W^2 \tilde{C}'_0(0; M_W, m_\alpha), \\
(h) &= -2m_\alpha^2 \tilde{C}'_3(0; M_W, m_\alpha),
\end{aligned}$$

$$D = F_1(-\not{q}q^\mu P_L) = \{c\} + \{d\} + \{e\} + \{h\} : \quad (4)$$

$$\begin{aligned}
\{c\} &= \frac{8}{3}M_W^2[\tilde{C}_1(0; m_\alpha, M_W) + \tilde{C}_2(0; m_\alpha, M_W)], \\
\{d\} &= \frac{4}{3}m_\alpha^2[\tilde{C}_1(0; m_\alpha, M_W) + \tilde{C}_2(0; m_\alpha, M_W)], \\
\{e\} &= M_W^2[\tilde{C}_0(0; M_W, m_\alpha) + 4\tilde{C}_2(0; M_W, m_\alpha) + 4\tilde{C}_1(0; M_W, m_\alpha)], \\
\{h\} &= m_\alpha^2[2\tilde{C}_2(0; M_W, m_\alpha) + 2\tilde{C}_1(0; M_W, m_\alpha) + \frac{1}{2}\tilde{C}_0(0; M_W, m_\alpha)],
\end{aligned}$$

The definitions and the explicit forms of the Green functions  $\tilde{B}'_1$ ,  $\tilde{C}_1$ ,  $\tilde{C}_2$ ,  $\tilde{C}'_0$ ,  $\tilde{C}'_3$ ,  $\tilde{B}'_0$ ,  $\tilde{C}_0$  can be found in the Appendix.

Only (a-d) graphs of the fig. 3 of ref. [9] contribute to the gauge invariant quarks-gluon vertex in the form factor  $F_g$ :

$$E = F_g(q^2\gamma^\mu P_L) = F_g(-\not{q}q^\mu P_L), \quad (5)$$

$$\begin{aligned}
F_g(q^2\gamma^\mu P_L) &= 2M_W^2\left\{\frac{1}{2}\left(1 + \frac{1}{2}\frac{m_\alpha^2}{M_W^2}\right)\tilde{B}'_1(0; m_\alpha, M_W) + \tilde{C}_2(0; m_\alpha, M_W) + \tilde{C}_1(0; m_\alpha, M_W) \right. \\
&+ 2\tilde{C}'_3(0; m_\alpha, M_W) - m_\alpha^2\tilde{C}'_0(0; m_\alpha, M_W) - \frac{1}{2}\frac{m_\alpha^2}{M_W^2}[m_\alpha^2\tilde{C}'_0(0; m_\alpha, M_W) - 2\tilde{C}'_3(0; m_\alpha, M_W) \\
&\quad \left. - \tilde{C}_1(0; m_\alpha, M_W) - \tilde{C}_2(0; m_\alpha, M_W)]\right\},
\end{aligned}$$

$$F_g(-\not{q}q^\mu P_L) = 2M_W^2\left(2 + \frac{m_\alpha^2}{M_W^2}\right)[\tilde{C}_1(0; m_\alpha, M_W) + \tilde{C}_2(0; m_\alpha, M_W)].$$

Knowing  $B_1, B_2, C, D$  functions, we construct the gauge invariant linear combinations  $X, Y, Z$  [8]:

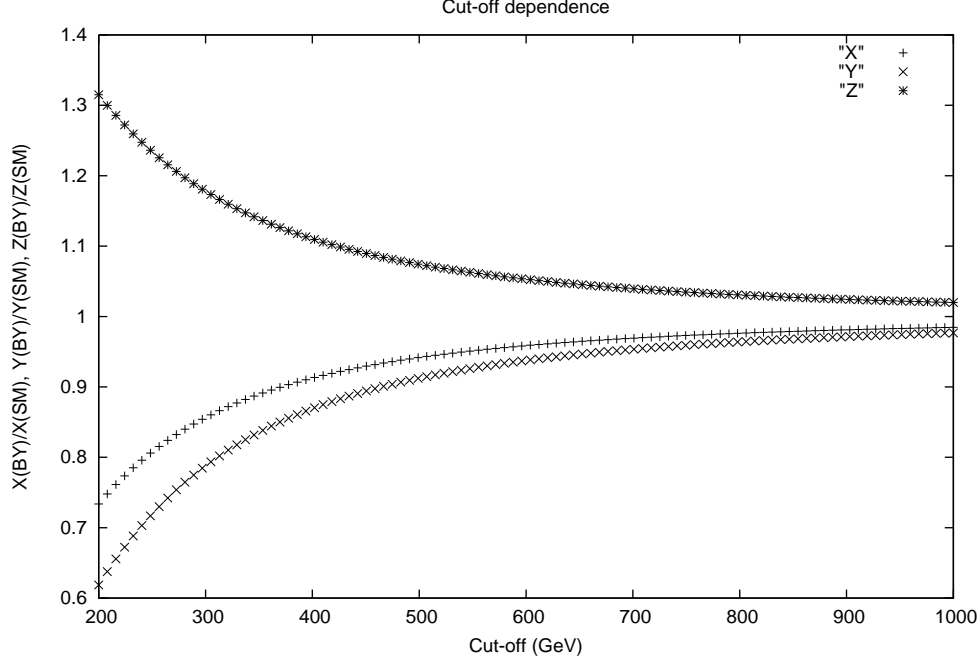
$$X = C - 4B_1, \quad Y = C - B_2, \quad Z = C + \frac{1}{4}D,$$

which appear in the amplitudes of the suppressed electroweak processes [13]. The  $B_1$  and  $B_2$  functions are almost equal for small lepton masses.

Numerical evaluations and conclusions are given in the next chapter.

### III. CONCLUSIONS

We can accomplish the aim of this paper to verify our SM results with the standard formulas (see the Appendix) and then to compare all the results with the functions of the BY theory containing the Lorentz and gauge invariant cut-off  $\Lambda$ .



**Fig. 1: Cut-off ( $\Lambda$ ) dependence of the quotients  $X^{BY}/X^{SM}$ ,  $Y^{BY}/Y^{SM}$  and  $Z^{BY}/Z^{SM}$ ; **X,Y,Z** are defined as  $X = X(x_t) - X(x_u)$  with parameters  $m_u = 3MeV$ ,  $m_t = 172GeV$ ,  $M_W = 80.4GeV$ .**

We verified that for any function  $F = \{B_1, B_2, C, D, E\}$  the following relation is valid  $\lim_{\Lambda \rightarrow \infty} F^{BY}(\Lambda) = F^{SM}$ .

The following definition is introduced in tables I and II:  $\Delta X(x_u, x_t) \equiv X(x_t) - X(x_u)$ ,  $x_t \equiv (\frac{m_t}{M_W})^2$  and similarly for other functions because these combinations appear in the amplitudes after applying the unitarity of the quark mixing matrix [9]. The functions without superscripts are the standard ones [8], and those with  $SM$  superscripts are the SM functions evaluated in this paper, whereas those with  $BY$  superscripts are the functions

TABLE I: Gauge invariant X,Y,Z,E functions evaluated with  $m_u = 3MeV, m_c = 1.3GeV$ ,  $M_W = 80.4GeV$ ,  $\Lambda = 326GeV$  and  $\sin^2 \Theta_W = 0.23$ .

$\Delta X^{SM}(x_u, x_c)$	$\Delta X^{BY}(x_u, x_c)$	$\Delta X(x_u, x_c)$	$[\frac{\Delta X(x_u, x_c)^{SM}}{\Delta X(x_u, x_c)}]^2$	$[\frac{\Delta X(x_u, x_c)^{BY}}{\Delta X(x_u, x_c)^{SM}}]^2$	$[\frac{\Delta X(x_u, x_c)^{BY}}{\Delta X(x_u, x_c)}]^2$
$1.5545 \times 10^{-3}$	$1.5487 \times 10^{-3}$	$1.5528 \times 10^{-3}$	1.0022	0.9925	0.9947
$\Delta Y^{SM}(x_u, x_c)$	$\Delta Y^{BY}(x_u, x_c)$	$\Delta Y(x_u, x_c)$	$[\frac{\Delta Y(x_u, x_c)^{SM}}{\Delta Y(x_u, x_c)}]^2$	$[\frac{\Delta Y(x_u, x_c)^{BY}}{\Delta Y(x_u, x_c)^{SM}}]^2$	$[\frac{\Delta Y(x_u, x_c)^{BY}}{\Delta Y(x_u, x_c)}]^2$
$1.3244 \times 10^{-4}$	$1.2695 \times 10^{-4}$	$1.3053 \times 10^{-4}$	1.0294	0.9188	0.9458
$\Delta Z^{SM}(x_u, x_c)$	$\Delta Z^{BY}(x_u, x_c)$	$\Delta Z(x_u, x_c)$	$[\frac{\Delta Z(x_u, x_c)^{SM}}{\Delta Z(x_u, x_c)}]^2$	$[\frac{\Delta Z(x_u, x_c)^{BY}}{\Delta Z(x_u, x_c)^{SM}}]^2$	$[\frac{\Delta Z(x_u, x_c)^{BY}}{\Delta Z(x_u, x_c)}]^2$
-1.3492	-1.3492	-1.3496	0.9994	1.0000	0.9995
$\Delta E^{SM}(x_u, x_c)$	$\Delta E^{BY}(x_u, x_c)$	$\Delta E(x_u, x_c)$	$[\frac{\Delta E(x_u, x_c)^{SM}}{\Delta E(x_u, x_c)}]^2$	$[\frac{\Delta E(x_u, x_c)^{BY}}{\Delta E(x_u, x_c)^{SM}}]^2$	$[\frac{\Delta E(x_u, x_c)^{BY}}{\Delta E(x_u, x_c)}]^2$
-8.0931	-8.0931	-8.0950	0.9995	1.0000	0.9995

TABLE II: Gauge invariant X,Y,Z,E functions evaluated with  $m_u = 3MeV, m_t = 172GeV$ ,  $M_W = 80.4GeV$ ,  $\Lambda = 326GeV$  and  $\sin^2 \Theta_W = 0.23$ .

$\Delta X^{SM}(x_u, x_t)$	$\Delta X^{BY}(x_u, x_t)$	$\Delta X(x_u, x_t)$	$[\frac{\Delta X(x_u, x_t)^{SM}}{\Delta X(x_u, x_t)}]^2$	$[\frac{\Delta X(x_u, x_t)^{BY}}{\Delta X(x_u, x_t)^{SM}}]^2$	$[\frac{\Delta X(x_u, x_t)^{BY}}{\Delta X(x_u, x_t)}]^2$
1.6111	1.4098	1.5777	1.0428	0.7657	0.7984
$\Delta Y^{SM}(x_u, x_t)$	$\Delta Y^{BY}(x_u, x_t)$	$\Delta Y(x_u, x_t)$	$[\frac{\Delta Y(x_u, x_t)^{SM}}{\Delta Y(x_u, x_t)}]^2$	$[\frac{\Delta Y(x_u, x_t)^{BY}}{\Delta Y(x_u, x_t)^{SM}}]^2$	$[\frac{\Delta Y(x_u, x_t)^{BY}}{\Delta Y(x_u, x_t)}]^2$
1.0595	0.8632	1.0261	1.0661	0.6637	0.7076
$\Delta Z^{SM}(x_u, x_t)$	$\Delta Z^{BY}(x_u, x_t)$	$\Delta Z(x_u, x_t)$	$[\frac{\Delta Z(x_u, x_t)^{SM}}{\Delta Z(x_u, x_t)}]^2$	$[\frac{\Delta Z(x_u, x_t)^{BY}}{\Delta Z(x_u, x_t)^{SM}}]^2$	$[\frac{\Delta Z(x_u, x_t)^{BY}}{\Delta Z(x_u, x_t)}]^2$
-1.2658	-1.4630	-1.5473	0.6693	1.3358	0.8940
$\Delta E^{SM}(x_u, x_t)$	$\Delta E^{BY}(x_u, x_t)$	$\Delta E(x_u, x_t)$	$[\frac{\Delta E(x_u, x_t)^{SM}}{\Delta E(x_u, x_t)}]^2$	$[\frac{\Delta E(x_u, x_t)^{BY}}{\Delta E(x_u, x_t)^{SM}}]^2$	$[\frac{\Delta E(x_u, x_t)^{BY}}{\Delta E(x_u, x_t)}]^2$
-12.7506	-12.8194	-13.3355	0.9142	1.0108	0.9241

attributed to the  $BY$  theory with  $\Lambda$ . One can notice that the functions for the c-quark agree for our SM outputs and the standard functions [8, 9] (table I), but there are disagreements for the t-quark for  $D$  and  $E$  functions (table II). We check our SM formulas for  $D$  and  $E$  through current conservation, hence these standard functions [9] are not accurate for the heaviest t-quark. The vertex contributions for  $\Gamma^Z$  agree perfectly graph by graph with the Inami-Lim [9] (A.1)-(A.3) results, but their sum is an approximation [12].

As expected, a comparison between the SM ( $\Lambda = \infty$ ) and the  $BY$  theory ( $\Lambda = 326GeV$ ) functions reveals a difference of roughly 10 - 30 %, thus opening the possibility to discover

the deviations from the SM for the suppressed electroweak processes and meson mixings, provided that the hadron matrix elements can be calculated with a sufficient precision and the measurements have small systematic and statistical errors.

## Appendix

For the purpose of comparison with our results, we give the standard functions widely used in the literature [8]

$$\begin{aligned}
B(x) &= \frac{1}{4} \left[ \frac{x}{1-x} + \frac{x \ln x}{(x-1)^2} \right], \\
C(x) &= \frac{x}{8} \left[ \frac{x-6}{x-1} + \frac{3x+2}{(x-1)^2} \ln x \right], \\
D(x) &= -\frac{4}{9} \ln x + \frac{-19x^3 + 25x^2}{36(x-1)^3} + \frac{x^2(5x^2 - 2x - 6)}{18(x-1)^4} \ln x, \\
E(x) &= -\frac{2}{3} \ln x + \frac{x^2(15 - 16x + 4x^2)}{6(1-x)^4} \ln x + \frac{x(18 - 11x - x^2)}{12(1-x)^3}.
\end{aligned}$$

Integrals appearing in  $B_1, B_2, C$  look like:

$$\begin{aligned}
g_1^{SM;BY}(m_1, m_2, m_3) &\equiv \int_0^{\infty; \Lambda} dx x [x + m_1^2]^{-1} [x + m_2^2]^{-1} [x + m_3^2]^{-2}, \\
f_1^{SM;BY}(m_1, m_2, m_3) &\equiv \int_0^{\infty; \Lambda} dx x^2 [x + m_1^2]^{-1} [x + m_2^2]^{-1} [x + m_3^2]^{-2}, \\
L^{SM;BY}(m_1, m_2) &\equiv -2 \int_0^{\infty; \Lambda} dq q^3 [q^2 + m_1^2]^{-2} [q^2 + m_2^2]^{-1},
\end{aligned}$$

and the explicit forms for the SM are:

$$\begin{aligned}
g_1^{SM}(m_1, m_2, m_3) &= (- (m_1^2(m_2^2 - m_3^2)^2 \ln m_1^2) + m_2^2(m_1^2 - m_3^2)^2 \ln m_2^2 - (m_1^2 - m_2^2) \\
&\times ((m_1^2 - m_3^2)(m_2^2 - m_3^2) + (m_1^2 m_2^2 - m_3^4) \ln m_3^2)) / ((m_1^2 - m_2^2)(m_1^2 - m_3^2)^2 (m_2^2 - m_3^2)^2), \\
f_1^{SM}(m_1, m_2, m_3) &= (m_1^4(m_2^2 - m_3^2)^2 \ln m_1^2 - m_2^4(m_1^2 - m_3^2)^2 \ln m_2^2 + (m_2^2 - m_1^2) m_3^2 \\
&\times ((m_1^2 - m_3^2)(m_3^2 - m_2^2) + (-2m_1^2 m_2^2 + (m_1^2 + m_2^2) m_3^2) \ln m_3^2)) / \\
&((m_1^2 - m_2^2)(m_1^2 - m_3^2)^2 (m_2^2 - m_3^2)^2),
\end{aligned}$$



$$L^{SM}(m_1, m_2) = \frac{1}{m_2^2} \left( \frac{m_1^2}{m_2^2} - 1 \right)^{-2} \left( 1 - \frac{m_1^2}{m_2^2} + \ln \frac{m_1^2}{m_2^2} \right),$$

$$L^{BY}(m_1, m_2) = -(\Lambda^2(m_1^2 - m_2^2) + m_2^2(\Lambda^2 + m_1^2) \left( \ln \frac{m_2^2}{m_1^2} + \ln \frac{\Lambda^2 + m_1^2}{\Lambda^2 + m_2^2} \right)) / ((\Lambda^2 + m_1^2)(m_1^2 - m_2^2)^2).$$

We do not show the analogous lengthy analytic expressions for other integrals with the cut-off. The analytic forms are checked with the numerically evaluated integrals to arbitrary precision.

The following definitions of the real parts of Green functions have been employed

$$(\frac{\partial \tilde{B}(k^2)}{\partial k^2} \equiv \tilde{B}'(k^2)):$$

$$\begin{aligned} \frac{i}{16\pi^2} \tilde{B}_0(k^2; m_1, m_2) &\equiv (2\pi)^{-4} \int d^4 q (q^2 - m_1^2)^{-1} ((q+k)^2 - m_2^2)^{-1}, \\ \frac{i}{16\pi^2} k_\mu \tilde{B}_1(k^2; m_1, m_2) &\equiv (2\pi)^{-4} \int d^4 q q_\mu (q^2 - m_1^2)^{-1} ((q+k)^2 - m_2^2)^{-1}, \\ \tilde{B}_1(0; m_1, m_2) &= \frac{1}{2} (-\tilde{B}_0(0, m_1, m_2) + (m_2^2 - m_1^2) \tilde{B}'_0(0, m_1, m_2)), \\ \tilde{B}'_1(0; m_1, m_2) &= \frac{1}{2} (-\tilde{B}'_0(0; m_1, m_2) + \frac{1}{2} (m_2^2 - m_1^2) \tilde{B}''_0(0; m_1, m_2)). \end{aligned}$$

The real parts of these Green functions, denoted by the superscripts SM and BY for the evaluation without and with the UV cut-off  $\Lambda$  respectively, turn out to be:

$$\begin{aligned} \tilde{B}_0^{SM}(0; m_1, m_2) &= \Delta_{UV} - x \ln x / (x - 1), \quad x = \frac{m_1^2}{m_2^2}, \quad \Delta_{UV} \equiv UV \text{ infinity}, \\ \tilde{B}'^{SM}_0(0; m_1, m_2) &= \frac{1}{2} \frac{m_1^2 + m_2^2}{(m_1^2 - m_2^2)^2} - \frac{m_1^2 m_2^2}{(m_2^2 - m_1^2)^3} \ln \frac{m_2^2}{m_1^2}, \\ \tilde{B}''^{SM}_0(0; m_1, m_2) &= [(m_1^2 - m_2^2)(m_1^4 + 10m_1^2 m_2^2 + m_2^4) + 6m_1^2 m_2^2 (m_1^2 + m_2^2) \ln \frac{m_2^2}{m_1^2}] / [3(m_1^2 - m_2^2)^5], \end{aligned}$$

$$\begin{aligned} \tilde{B}_0^{BY}(0; m_1, m_2) &= \int_0^{\Lambda^2} dy \frac{y}{(y + m_1^2)(y + m_2^2)} \\ &= (m_1^2 \ln \frac{\Lambda^2 + m_1^2}{m_1^2} - m_2^2 \ln \frac{\Lambda^2 + m_2^2}{m_2^2}) / (m_1^2 - m_2^2), \end{aligned}$$

$$\frac{\partial \tilde{B}_0^{BY}}{\partial k^2}(0; m_1, m_2) = \frac{1}{2} \left( \frac{\partial \hat{B}_0^{BY}}{\partial k^2}(0; m_1, m_2) + \frac{\partial \hat{B}_0^{BY}}{\partial k^2}(0; m_2, m_1) \right),$$

$$\begin{aligned}
\frac{\partial \hat{B}_0^{BY}}{\partial k^2}(0; m_1, m_2) &= m_2^2 \int_0^{\Lambda^2} dy \frac{y}{(y + m_1^2)(y + m_2^2)^3} \\
&= [\Lambda^2(-2m_1^2 m_2^4 + 2m_2^6 + \Lambda^2(m_2^4 - m_1^4)) + 2m_1^2 m_2^2(\Lambda^2 + m_2^2)^2 \\
&\quad \times (\ln \frac{m_1^2}{m_2^2} - \ln \frac{\Lambda^2 + m_1^2}{\Lambda^2 + m_2^2})] / [2(\Lambda^2 + m_2^2)^2(m_2^2 - m_1^2)^3],
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 \tilde{B}_0^{BY}}{\partial (k^2)^2}(0; m_1, m_2) &= \frac{1}{2} \left( \frac{\partial^2 \hat{B}_0^{BY}}{\partial (k^2)^2}(0; m_1, m_2) + \frac{\partial^2 \hat{B}_0^{BY}}{\partial (k^2)^2}(0; m_2, m_1) \right), \\
\frac{\partial^2 \hat{B}_0^{BY}}{\partial (k^2)^2}(0; m_1, m_2) &= 2m_2^2 \int_0^{\Lambda^2} dy \frac{y(m_2^2 - y)}{(y + m_1^2)(y + m_2^2)^5}.
\end{aligned}$$

Let us finally define and evaluate the necessary vertex Green functions:

$$\begin{aligned}
\frac{i}{16\pi^2} \tilde{C}_0(k^2; m_1, m_2) &\equiv (2\pi)^{-4} \int d^4 q [(q+k)^2 - m_1^2]^{-1} [q^2 - m_1^2]^{-1} [(q+k/2)^2 - m_2^2]^{-1}, \\
\frac{i}{16\pi^2} k_\mu \tilde{C}_1(k^2; m_1, m_2) &\equiv (2\pi)^{-4} \int d^4 q q_\mu [(q+k)^2 - m_1^2]^{-1} [q^2 - m_1^2]^{-1} [(q+k/2)^2 - m_2^2]^{-1}, \\
\frac{i}{16\pi^2} [k_\mu k_\nu \tilde{C}_2(k^2; m_1, m_2) + g_{\mu\nu} \tilde{C}_3(k^2; m_1, m_2)] &= (2\pi)^{-4} \int d^4 q q_\mu q_\nu [(q+k)^2 - m_1^2]^{-1} [q^2 - m_1^2]^{-1} [(q+k/2)^2 - m_2^2]^{-1}.
\end{aligned}$$

It is easy to verify that  $\tilde{C}_1(k^2; m_1, m_2) = -\frac{1}{2} \tilde{C}_0(k^2; m_1, m_2)$  and:

$$\begin{aligned}
\tilde{C}_2(k^2; m_1, m_2) &= \frac{1}{3k^2} [\tilde{B}_0(k^2/4; m_1, m_2) + 2\tilde{B}_1(k^2/4; m_1, m_2) - (m_1^2 - k^2) \tilde{C}_0(k^2; m_1, m_2)], \\
\tilde{C}_3(k^2; m_1, m_2) &= \frac{1}{3} [\frac{1}{2} \tilde{B}_0(k^2/4; m_1, m_2) - \frac{1}{2} \tilde{B}_1(k^2/4; m_1, m_2) + (m_1^2 - k^2/4) \tilde{C}_0(k^2; m_1, m_2)].
\end{aligned}$$

We need Green functions evaluated at  $k^2 = 0$ , thus the following values of  $\tilde{C}_0$  and  $\tilde{C}'_0$  should be calculated:

$$\begin{aligned}
\tilde{C}_0^{SM;BY}(0; m_1, m_2) &= -2 \int_0^{\infty; \Lambda} dq q^3 (q^2 + m_1^2)^{-2} (q^2 + m_2^2)^{-1}, \\
\tilde{C}'_0^{SM;BY}(0; m_1, m_2) &= -\frac{1}{2} \int_0^{\infty; \Lambda} dq q^3 (2m_1^2 + q^2) (q^2 + m_1^2)^{-4} (q^2 + m_2^2)^{-1}.
\end{aligned}$$

Straightforward integration yields:

$$\begin{aligned}
\tilde{C}_0^{SM}(0; m_1, m_2) &= (m_2^2 - m_1^2 + m_2^2 \ln \frac{m_1^2}{m_2^2}) / (m_1^2 - m_2^2)^2, \\
\tilde{C}_0^{'SM}(0; m_1, m_2) &= -[5m_1^6 - 9m_1^2 m_2^4 + 4m_2^6 - 6m_1^2 m_2^2 (2m_1^2 - m_2^2) \ln \frac{m_1^2}{m_2^2}] / [24m_1^2 (m_1^2 - m_2^2)^4], \\
\tilde{C}_0^{BY}(0; m_1, m_2) &= -[(m_1^2 - m_2^2 + m_2^2 \ln \frac{m_2^2}{m_1^2}) / (m_1^2 - m_2^2)^2 + (-m_1^4 + m_1^2 m_2^2 + (\Lambda^2 + m_1^2) m_2^2 \\
&\quad \times \ln(\Lambda^2 + m_1^2) - (\Lambda^2 + m_1^2) m_2^2 \ln(\Lambda^2 + m_2^2)) / ((\Lambda^2 + m_1^2) (m_1^2 - m_2^2)^2)].
\end{aligned}$$

Lengthy analytical expressions for integrals in  $\frac{\partial^2 \tilde{B}_0^{BY}}{\partial (k^2)^2}(0; m_1, m_2)$  and  $\tilde{C}_0^{'BY}(0; m_1, m_2)$  are not given. Displayed functions suffice to fix  $\tilde{C}_2$  appearing in the amplitudes:

$$\begin{aligned}
\tilde{C}_2(k^2; m_1, m_2) &= \frac{a_{-1}}{k^2} + a_0 + \mathcal{O}(k^2), \\
\tilde{C}_2(0; m_1, m_2) \equiv a_0 &= \frac{1}{3} [\frac{1}{4} \tilde{B}_0'(0; m_1, m_2) + \frac{1}{2} \tilde{B}_1'(0; m_1, m_2) + \tilde{C}_0(0; m_1, m_2) \\
&\quad - m_1^2 \tilde{C}_0'(0; m_1, m_2)].
\end{aligned}$$

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